TRINITY COLLEGE

Year 12 Applications Units 3 & 4
Test 6 2016

Calculator Assumed

Project Networks and Assignment Problems

STUDENT'S NAME SOLNS

DATE: Friday 2nd September **TIME:** 50 minutes **MARKS**: 51

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (2 marks)

The Spicy Spoon restaurant has four payment counters. There are four people available for service. The cost of assigning each person to each counter is given in the following table.

Person	Job			
	1	2	3	4
A	1	8	15	22
B	13	18	23	28
C	13	18	23	28
D	19	23	27	31

After minimising the rows and then the columns the following matrix was produced.

$$\begin{bmatrix} 0 & 2 & 4 & 7 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 \end{bmatrix}$$

Determine the optimal job allocation(s) to this problem and the cost associated.

SOLN 1

A1
B2
C3
D4

SOLN 2

A1
B3
C2
D4

1 + 18 + 23 + 31 = $73

need both solutions and cost.
-1 mark for anything missing

2. (14 marks)

Four tasks, A, B, C, and D must be completed. Four workers, James, Kane, Luke and Mick, will each do one task. Table 1 shows the time, in minutes, that each person would take to complete each of the four tasks.

	Worker			
Task	James	Kane	Luke	Mick
A	26	21	22	25
B	31	26	21	38
C	29	26	20	27
D	38	26	26	25

(a) Complete the cost matrix. [1]

$$\begin{bmatrix} 26 & 21 & 22 & 25 \\ 31 & 26 & 21 & 38 \\ 29 & 26 & 20 & 27 \\ 38 & 26 & 26 & 25 \end{bmatrix}$$

(b) By subtracting the smallest number in each row, complete the row matrix. [2]

$$\begin{bmatrix} 5 & 0 & 1 & 4 \\ 10 & 5 & 0 & 17 \\ 9 & 6 & 0 & 7 \\ 13 & 1 & 1 & 0 \end{bmatrix}$$

-1 per mistake

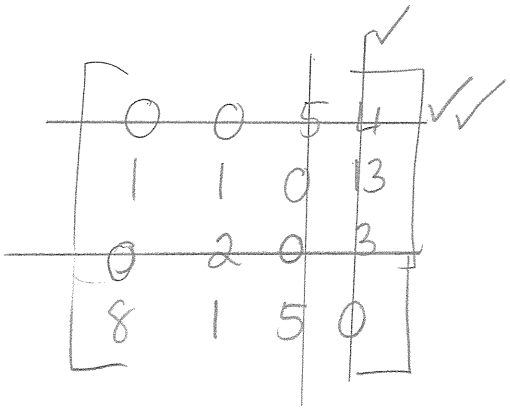
(c) By subtracting the smallest number in each column, complete the column matrix. [2]

$$\begin{bmatrix} 0 & 0 & 1 & 4 \\ 5 & 5 & 0 & 17 \\ 4 & 6 & 0 & 7 \\ 8 & 1 & 1 & 0 \end{bmatrix}$$

(d) Using a suitable method, show that the matrix in (c) is not suitable to determine the optimal allocation. Explain your answer. [2]

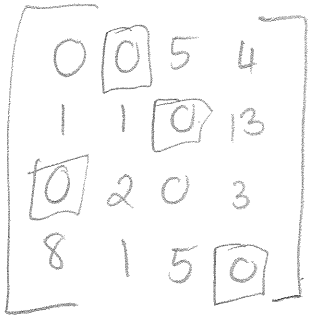
- Any 3 lines ✓
- lines < rows
∴ not optimal soln ✓

(e) Complete the steps necessary to produce a matrix from which the optimal allocation of tasks can be made. [3]



lines = row ∴ optimal soln

(f) Write the name of the person that should do each task for the optimal allocation of tasks. [2]



Task	Worker	
A	Kane	21
B	Luke ✓	21
C	James	29
D	Mick ✓	25

(g) Determine the minimum time needed to complete the four tasks if they are done;

(i) consecutively (one after the other) [1]

21 + 21 + 29 + 25 = 96 minutes ✓

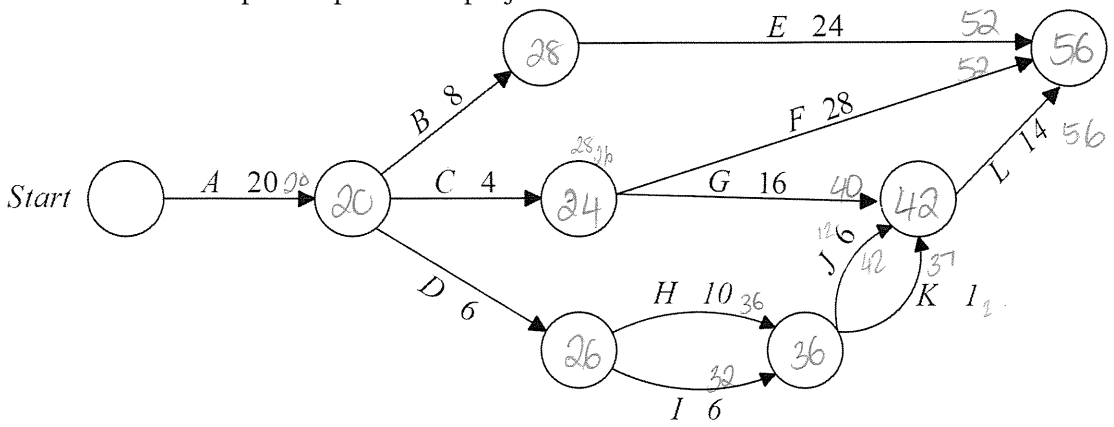
(ii) simultaneously [1]

Longest time is 29

∴ 29 minutes

3. (11 marks)

The following project network gives the tasks and times, in days, that are required to be undertaken to complete a particular project.



(a) State the minimum completion time for the project. [1]

56 days ✓

(b) State the critical path(s). [2]

A D H J L ✓✓

(c) Complete the precedent table below. [2]

Activity	Immediately Preceded by:
A	—
B	A
C	A
D	A
E	B
F	C

Activity	Immediately Preceded by:
G	C
H	D
I	D
J	H I
K	H J
L	E F L

−1 per error.

(d) How long can task C be delayed without affecting the minimum completion time? [1]

A delay of 2 days

A C I 2 G L = 56

(e) Tasks J and K take twice as long as stated in the project network. What effect will this have? [2]

K will have no effect.

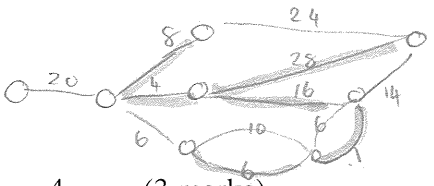
J will increase the minimum comp. time by 6 days but will not change the critical path

(f) How long will it take to complete all ~~six~~ 12 tasks if;

(i) one person is allocated to project? [1]

143 days

(ii) two people are allocated to the project? [2]



Person 1 | Person 2

critical path 56 | 20 wait

E + K 24+1 | B C I 18 | 38

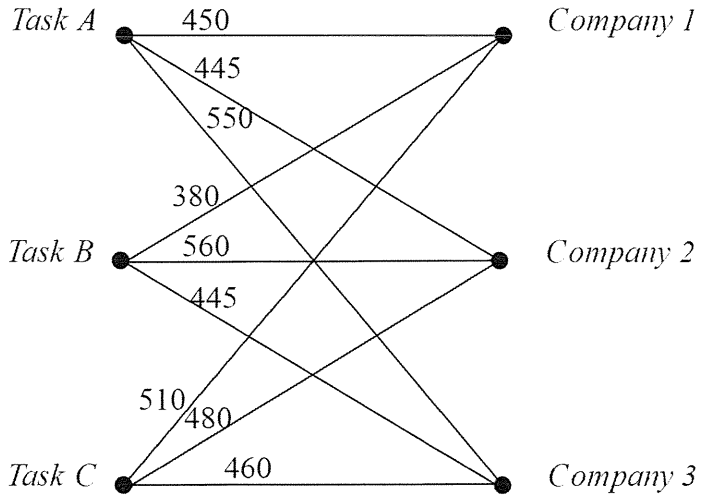
F G 44 | 82

81 | 82

82 days ✓✓

4. (3 marks)

Represent the information shown in the following bipartite graph in a table.



2 marks if matrix drawn without headings

PERSON	Company 1	Company 2	Company 3
A	450	445	550
B	380	560	445
C	510	480	460

✓✓✓

5. (7 marks)

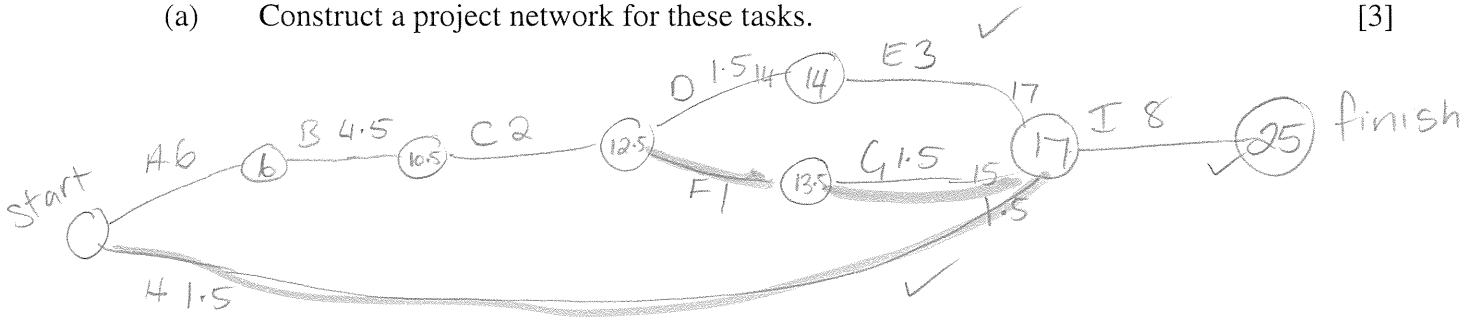
The successful preparation of a Year 12 Mathematics Examination requires the completion of the following tasks.

TASK	ACTIVITY	TIME (in hours)
A	Select exam questions	6
B	Do solutions to selected questions	4.5
C	Make necessary alterations	2
D	Chose the order of the questions	1.5
E	Type paper	3
F	Make cover sheet for exam	1
G	Print exam booklets	3
H	Set up the hall	1.5
I	Mark exam papers	8

The order in which these task should be completed is:

Tasks A and H have no predecessors
Task B must follow task A
Task B must be completed before task C
Tasks D and F must be after task C
Task E must follow task D
Task G must follow task F
Task I must be after tasks E, G and H

(a) Construct a project network for these tasks. [3]



(b) State the critical path and minimum completion time for this project. [2]

25 hours ✓ A B C D E I ✓

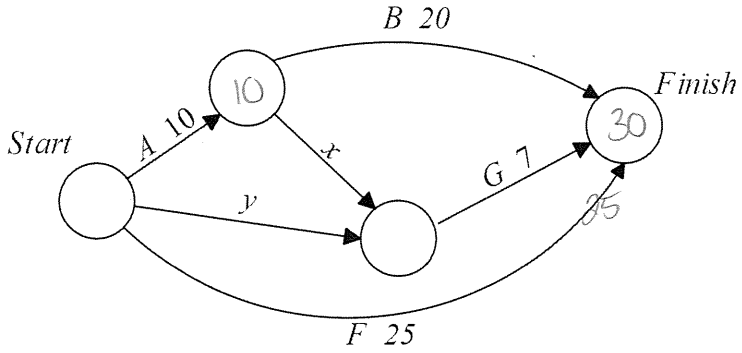
(c) Marking the exam booklets turned out to be a bigger job than expected. How much extra time could be devoted to this task without increasing the minimum time required? [1]

0, Task I on critical path ∴ no extra time

(d) How many teachers need to work on the exam to ensure it is completed in the minimum number of hours? [1]

1 teacher critical path.
2nd teacher F, G, H ∴ 2 teachers

6. (2 marks)



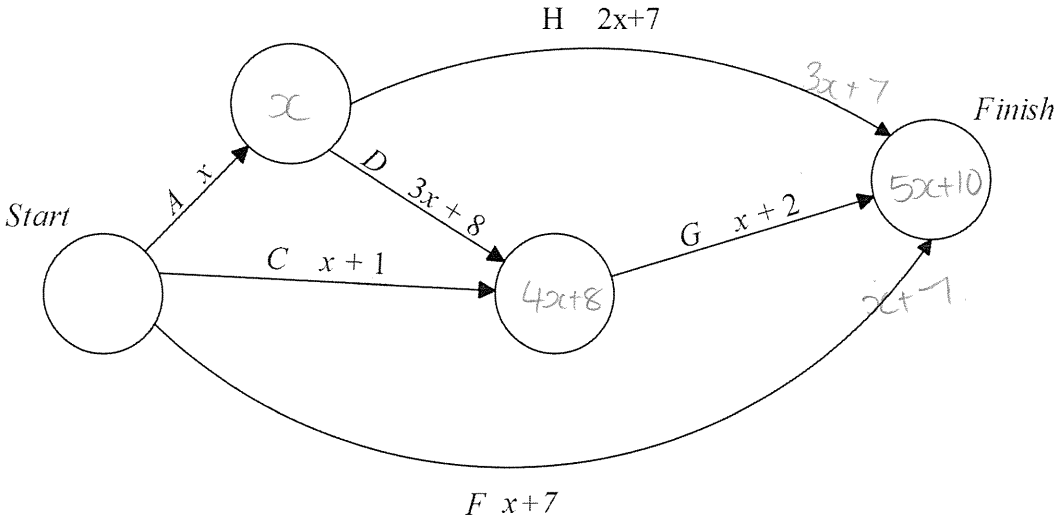
Given that the critical path is AB, determine the possible value(s) of x

$x + 7 < 20$

$x < 13$

$\therefore \quad 0 < x < 13$ ✓ ✓

7. (3 marks)



Given that $x > 0$ determine the critical path and minimum completion time, in terms of x.

CG, $5x+10$ ✓ ✓

8. (9 marks)

Consider the following 3 by 4 table, where x is an integer > 7.

	Worker		
Task	1	2	3
A	$x + 4$ 2x	$3x + 4$ *	x
B	x 2x+4	$x - 2$	$x + 1$
C	$2x - 1$ * x+5	$x - 3$	$x - 1$
D	$x + 6$ 2x-2	$x + 2$	$x + 6$ *

(a) What changes need to be made to the table above to make it suitable to use the Hungarian Algorithm? [1]

Add a dummy column of zeros ✓

(b) Which expression is the highest value? [1]

3x+4 ✓

(c) The table above is being used to maximise the allocation. Complete the first column with the expressions required in the first step of maximising the Hungarian Algorithm. [3]

-1 per error

(d) The Hungarian Algorithm has been used to create the matrix below. Determine the optimal allocation of tasks and the cost (in terms of x) associated. [2]

W Task
1 C
2 A
3 D ✓

$$\begin{bmatrix} 2x & \boxed{0} & 2x+4 & 2x+3 \\ 1 & 3 & 0 & \boxed{0} \\ \boxed{0} & x+2 & x & x-2 \\ 0 & 4 & \boxed{0} & 5 \end{bmatrix}$$

3x+4+2x-1+x+6
= 6x+9 ✓

(e) One of the 3 workers will need to complete a second task. Which worker should the company pick? Justify your choice. [2]

Task B hasn't been assigned

1	2	3
2x+4	x-2	x+1

Worker 1 will maximise task.